Chapter 9
The Economics of Crime

The Optimal Enforcement Model

These notes present a formal version of the economic model of criminal enforcement that is used to derive the results in the text. The model uses the following notation:

g = offender's dollar gain from committing a crime;
h = harm to society per crime;
p = probability of apprehension;
$k(p)$ = cost of apprehension, $k'>0$, $k''>0$;
f = fine upon conviction;
t = prison term upon conviction;
c = unit cost of prison to offenders;
_ = unit cost of prison to society.

Assume that g is distributed across potential offenders according to the distribution function $Z(g)$, where Z'_z is the density function.

An offender will commit a crime, given g, if

$$g > p(f+ct). \tag{9.1}$$

Thus, for any set of policy variables, (p,f,t), the expected number of crimes across offenders (the crime rate) is $1–Z(p(f+ct))$. It follows that the crime rate is decreasing in each of these variables, reflecting the deterrent effect of law enforcement.

The social problem is to choose the policy variables to maximize social welfare, given by the sum of the gains to offenders minus the costs of crime to victims and enforcement costs:

$$W = \int_{p(f+ct)}^{\infty} (g - h - p\alpha t)dZ(g) - k(p). \tag{9.2}$$

As in the text, we first consider optimal enforcement when p is fixed. Later, we allow p to vary.

Probability of Apprehension is Fixed

Fines only. First consider the case of fines only. Setting t=0 in (9.2) and taking the derivative with respect to f yields the first order condition

$$(pf–h)z(pf)p = 0. \tag{9.3}$$

It follows that $f^*=h/p$, as shown in the text. Intuitively, the offender to should face a fine equal to the harm his action causes, inflated by the inverse of the probability that he will be caught.

Fines and prison. Since prison is costly, fines should be imposed to the maximum extent before prison is used. Thus, if w is the offender's wealth, the optimal prison term is zero if $h/p \geq w$, and $f^*=h/p$ as above. However, if $h/p>w$, it is optimal to set $f^*=w$ (i.e., the fine should be maximal), and $t>0$ may also be optimal, depending on whether the marginal social gain from the additional deterrence exceeds the marginal cost. Formally, taking the derivative of W with respect to t with $f=w$ yields

$$\frac{\partial W}{\partial t} = [h+p\_t-p(w+ct)]z(p(w+ct))pc - p\_[1-Z(p(w+ct))] \qquad (9.4)$$

where the first term is the net deterrence benefit from increasing t, and the second term is the marginal punishment cost. If this expression is positive when evaluated at $t=0$, then some prison is optimal. In that case, the optimal prison term solves (9.4) when set equal to zero.

Probability of Apprehension is Variable

Fines only. Based on the above logic, the fine should be set maximally before p is increased. Thus, $f^*=w,$ and the optimal p is the solution to the first order condition

$$\frac{\partial W}{\partial p} = (h-pw)z(pw)w - k'(p) = 0, \qquad (9.5)$$

or

$$(h-pw)z(pw)w = k'(p). \qquad (9.6)$$

It follows that at the optimum level of enforcement, $h>pw$, or the harm exceeds the expected punishment. Thus, there is some underdeterrence relative to the above model where p was fixed.

Fines and prison. The optimum in this case again involves a maximal fine, or $f^*=w$. Then maximize W with respect to t and p. Assuming an interior solution for each, we obtain the first order conditions

$$\frac{\partial W}{\partial t} = -[p(w+ct)-h-p\_t]z(\bullet)pc - p\_[1-Z(\bullet)] = 0 \qquad (9.7)$$

$$\frac{\partial W}{\partial p} = -[p(w+ct)-h-p\_t]z(\bullet)(w+ct) - \_t[1-Z(\bullet)] - k'(p) = 0. \qquad (9.8)$$